Visible Learning for Mathematics

Webinar #3: Surface Learning & Deep Learning

Spring, 2017

based on Visible Learning For Mathematics by John Hattie, Douglas Fisher, and Nancy Frey
Hopes & Dreams for Today

- Surface Learning – continued
  - Talk
  - Representation

- Deep Learning
  - Tasks
  - Talk (part I)
BE INTERESTED, NOT INTERESTING

• Presume positive intentions
• Practice productive dialogue skills*
• Take care of self and others
• Appropriate use of technology
• Give yourself a break professionally
“Hinge Point” is 0.40
## Effect Sizes

**Talk**
- Questioning – 0.48
- Classroom Discussion – 0.82
- Self-questioning - 0.64

**Tasks**
- Spaced vs. Mass Practice - 0.71
- Problem-Solving Teaching – 0.61
- Meta-Cognitive Strategies – 0.69

**Clarity**
- Teacher Clarity – 0.75
- Self-Reported Grades/Student Expectations – 1.44
- Classroom Cohesion – 0.53
Phases of Learning

Surface Learning
- Understand

Deep Learning
- Connect/Generalize
- Apply

Transfer Learning
- Understand
- Connect/Generalize
- Apply
Surface Learning

✓ Tasks: Low Difficulty & Low-Moderate Complexity

➢ Talk:
  ✓ Direct Instruction – 0.59
  ✓ Worked Examples – 0.57
    ▪ Number Talks
    ▪ Guided Questions
Were there any questions from the last webinar that we didn’t answer?
Talk for Surface Learning

Types of Math Talk:
• Direct Instruction ✓
• Worked Examples ✓
• Number Talks
• Guided Questions

Effect Sizes:
• Questioning – 0.48
• Classroom Discussion – 0.82
• Self-Verbalization & Self-questioning - 0.64

“Doing math is not eating bananas.”
Number Talks

- Brief, ongoing, daily routine that helps students develop computational fluency through the opportunity to share their thinking processes out loud.
- Usually done with mental calculations.

1. Teacher puts a problem on the board: 3,000 – 1,345
2. Students get some think time.
3. Students share their answers and their reasoning.
4. Students & teacher ask clarifying questions and discuss each solution strategy.

Effect Size for Classroom Discussion – 0.82
Number Talks

3,000 – 1,345

**Student A**
1345 + 5 = 1350
1350 + 50 = 1400
1400 + 600 = 2000
2000 + 1000 = 3000

**Student B**
3000 – 1000 = 2000
2000 - 300 = 1700
1700 – 40 = 1660
1660 – 5 = 1655

**Sample Questions:**
- Why did you add 5 first?
- Why did you start by subtracting 1000?
- Am I following your thinking?
- Did anyone else do it this way?
- Did anyone use a different strategy?
- How are these strategies the same?
- How are they different?
- Is one more efficient than the other? Why?
Starting Math Talks

Dot Talks

wodb.ca
Which One Doesn’t Belong

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = 4x + 3$</td>
<td>$y = -4x + 5$</td>
</tr>
<tr>
<td>$y = \frac{1}{4}x + 5$</td>
<td>$y = 4x - 5$</td>
</tr>
</tbody>
</table>
Think - (Pair) - Share

➢ Think: Have you used Number Talks (Math Talks) with students? What challenges did you encounter?
➢ Share: One challenge (& a solution if you found one)
Guided Questions

- Questions designed to help students make sense out of what is going on and guide them to draw conclusions on their own
- Give just enough information to help students get started or unstuck
- Should push student thinking forward
- Generally address the “why” more than the “what”

**Effect sizes:**
- Questioning: 0.48
- Metacognitive Strategies: 0.69
- Self-Verbalization & Self-Questioning – 0.64
Funneling vs. Focusing Questions

**Funneling Questions**
- Guide students down the path to find the answer
- Hint at an answer
- Tell you what to see

**Focusing Questions**
- Guide students to figure out next steps in THEIR process
- Build on student thinking
- Show you where to look
# Sample Questions

<table>
<thead>
<tr>
<th>Funneling Questions</th>
<th>Focusing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>How do you find the mean of the data? What about the median and the mode? What about the interquartile range?</td>
<td>What do you notice about the data? How would you describe them to someone? What makes you say that? What other ways might you be able to describe them?</td>
</tr>
<tr>
<td>How can I get rid of the 2? What do I have to do to the other side? What about the 4?</td>
<td>What do you think about when you see this equation? How do you want to solve it?</td>
</tr>
<tr>
<td>How do I find the area of this trapezoid? Do you see the rectangle and the triangles? I can just add them up. How can I find the area of the rectangle?</td>
<td>I want to know the area of this trapezoid, but I’m not sure how to find it. Any ideas? Where should we start?</td>
</tr>
<tr>
<td>Let’s add these fractions by finding the least common denominator. What’s the first step in finding the least common denominator?</td>
<td>What should we do with these fractions? [Student: “Add them.”] Why add them? [Student refers to word problem.] Okay, so how would you add them?</td>
</tr>
</tbody>
</table>

Figure 3.4, p. 92
## Question Types

<table>
<thead>
<tr>
<th>Question type</th>
<th>Description</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Students recall facts, definitions, or procedures.</td>
<td>When you write an equation, what does the equal sign tell you?</td>
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<tr>
<td></td>
<td></td>
<td>What is the formula for finding the area of a rectangle?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>What does the interquartile range indicate for a set of data?</td>
</tr>
<tr>
<td>Gathering information</td>
<td></td>
<td>As you drew that number line, what decisions did you make so that you could represent 7 fourths on it?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Can you show and explain more about how you used a table to find the answer to the Smartphone Plans task?</td>
</tr>
<tr>
<td>Probing thinking</td>
<td>Students explain, elaborate, or clarify their thinking, including articulating the steps in solution methods or the completion of a task.</td>
<td>It is still not clear how you figured out that 20 was the scale factor, so can you explain it another way?</td>
</tr>
</tbody>
</table>

*from Principles to Actions, NCTM 2014*
## Question Types

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<thead>
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<th>Question type</th>
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<th>Examples</th>
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</thead>
<tbody>
<tr>
<td>Making the mathematics visible</td>
<td>Students discuss mathematical structures and make connections among</td>
<td>What does your equation have to do with the band concert situation?</td>
</tr>
<tr>
<td></td>
<td>mathematical ideas and relationships.</td>
<td>How does that array relate to multiplication and division?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>In what ways might the normal distribution apply to this situation?</td>
</tr>
<tr>
<td>Encouraging reflection and justification</td>
<td>Students reveal deeper understanding of their reasoning and actions,</td>
<td>How might you prove that 51 is the solution?</td>
</tr>
<tr>
<td></td>
<td>including making an argument for the validity of their work.</td>
<td>How do you know that the sum of two odd numbers will always be even?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Why does plan A in the Smartphone Plans task start out cheaper but become more expensive in the long run?</td>
</tr>
</tbody>
</table>

*from Principles to Actions, NCTM 2014*
Think - (Pair)- Share

- What are some overall themes you noticed with these sample questions?
- Is there a question type that you’d like to include more often in your classroom?
Meta-Cognitive Strategies

- Having a plan for approaching a task
- Using appropriate skills and strategies to solve a problem
- Monitoring and noticing when a problem doesn’t make sense
- Self-assessing and self-correcting
- Evaluating progress toward completion of the task

Effect Size for Meta-Cognitive Strategies – 0.69
Encouraging Meta-Cognition

- Decrease the “What”, Increase the “Why” & “How” questions
- Teachers model Think-Alouds
- Use “I” statements (students recognize that they are the force acting upon & understanding the math)
- Direct instruction on the SMPs & problem solving strategies
Encouraging Meta-Cognition

Sample Sentence Frames:

- I wonder why....
- I know the word _____, but I don’t know what ____ & ____ mean
- I think/believe that ..... 
- I discovered from _____ that ______
- My idea is similar to ______ because ______
- My idea is different from _____ because _____
- My answer is reasonable because ....
- What strategies worked well for me? Why?
- What should I do next time?
- What questions do I have?
Vocabulary

- **Word Walls** – ongoing, organized display of key words that provides visual reference for students throughout a unit of study or term.
- **Graphic Organizers** – visual display that demonstrates relationships between words, facts, concepts, and/or ideas.

- **Effect Size of Vocabulary Programs** – 0.67
- **Classroom Discussion** – 0.82
- **SMP #6** Attend to precision.
- **Vocabulary is an indicator of overall understanding across content**
<table>
<thead>
<tr>
<th>Definition</th>
<th>Facts/Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polygon</td>
<td>A closed plane figure that has at least 3 sides that are straight</td>
</tr>
</tbody>
</table>

**Examples**
- Rectangle
- Triangle
- Hexagon

**Non-Examples**
- Oval

**Frayer Model**
Frayer Model

**My Definition**

A quadratic function is a function that can be written in the standard form

\[ f(x) = ax^2 + bx + c, \text{ where } a \neq 0. \]

**Example**

\[ f(x) = 3x^2 + 2x - 9 \]

\[ f(x) = (x-3)(x+2) = x^2 + 2x - 3x - 6 = x^2 - x - 6 \]

**Characteristics**

The graph of a quadratic function is a parabola.

Quadratic functions can also be written in vertex form or intercept form.

**Vocabulary word:**

**Quadratic Function**

**Non-Example**

\[ f(x) = 3x + 5 \]

\[ f(x) = 9x^3 + 2x^2 + 7 \]
Think - (Pair) - Share

➢ What are some vocabulary strategies that work well for you?
Representation in Mathematics

Concrete <----> Pictorial <----> Abstract

3x - 5y = 45
<table>
<thead>
<tr>
<th>Multiple Representations</th>
<th>What are teachers doing?</th>
<th>What are students doing?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Selecting tasks that allow students to decide which representations to use in making sense of the problems.</td>
<td>Using multiple forms of representations to make sense of and understand mathematics.</td>
</tr>
<tr>
<td></td>
<td>Allocating substantial instructional time for students to use, discuss, and make connections among representations.</td>
<td>Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations.</td>
</tr>
<tr>
<td></td>
<td>Introducing forms of representations that can be useful to students.</td>
<td>Making choices about which forms of representations to use as tools for solving problems.</td>
</tr>
<tr>
<td></td>
<td>Asking students to make math drawings or use other visual supports to explain and justify their reasoning.</td>
<td>Sketching diagrams to make sense of problem situations.</td>
</tr>
<tr>
<td></td>
<td>Focusing students’ attention on the structure or essential features of mathematical ideas that appear, regardless of the representation.</td>
<td>Contextualizing mathematical ideas by connecting them to real-world situations.</td>
</tr>
<tr>
<td></td>
<td>Designing ways to elicit and assess students’ abilities to use representations meaningfully to solve problems.</td>
<td>Considering the advantages or suitability of using various representations</td>
</tr>
</tbody>
</table>
Manipulatives for Surface Learning

- Concrete models student can use to show and work with representations of math concepts
- Hands-on, can be manipulated and moved
- Can be physical or virtual

Effect Size for Manipulatives – 0.50
- Must connect the representation with the manipulatives to the pictorial and abstract representations

\[ 1 + 4 = 5 \]
Manipulatives for Surface Learning

2x + 2 = 4
- 2   -2
2x = 2
2   2
x = 1

Illuminations.nctm.org
Think - (Pair)- Share

➢ What are some challenges (& solutions if you have them) that you’ve faced in using manipulatives with students?
Phases of Learning

- Surface Learning
  - Understand
- Deep Learning
  - Connect/Generalize
- Transfer Learning
  - Apply
Making connections, recognizing the relationships among ideas, describing patterns, engaging more actively and deliberately with math concepts in order to discover underlying structures, explaining and justifying mathematical ideas, making mathematical arguments...

**DEEP LEARNING**: Consolidating understanding of mathematical concepts & procedures and making connections among ideas – students plan, investigate & elaborate on their conceptual understandings and begin to make generalizations...
Tasks for Deep Learning

- Require greater cognitive demand
- Usually more open-ended
- Multiple solutions & multiple paths to solution
- allow students to move between concrete -> pictorial->abstract representations
Tasks for Deep Learning

Choose tasks that emphasize SMPs #3, 7, 8

| SMP #3: Construct viable arguments and critique the reasoning of others. |
| SMP #7: Look for and make use of structure. |
| SMP #8: Look for and express regularity in repeated reasoning. |
Tasks for Deep Learning

**Surface Learning Exercise:**
Find the circumference of a circle with a:
1. diameter of 8 inches
2. diameter of 30 feet
3. radius of 10 mm
4. radius of 40 cm

**Deep Learning Task:**
Design a tape measure that, when wrapped around a cylinder, could be used to directly read the diameter of the cylinder.

from *Visible Learning for Mathematics*, Hattie 2017
Tasks for Deep Learning

**Surface Learning Exercise:**
Order the following fractions by converting them to a common denominator.

\[
\begin{array}{ccc}
\frac{19}{15} & \frac{7}{12} & \frac{5}{6} \\
\frac{41}{30} & \frac{11}{15} &
\end{array}
\]

**Deep Learning Task:**
Draw an X on the number line where you think the fraction 11/8 should be. Explain why you put your X there. Draw and label a few others points to help you explain your reasoning.

from *Visible Learning for Mathematics*, Hattie 2017
Tasks for Deep Learning

A-SSE Increasing or Decreasing? Variation 1

Consider the expression

\[ \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \]

where \( R_1 \) and \( R_2 \) are positive.

Suppose we increase the value of \( R_1 \) while keeping \( R_2 \) constant. Does the value of the expression above increase, decrease, or stay the same? Explain in terms of the structure of the expression.

from Illustrativemathematics.org
Rich Tasks Should:

- Be complex enough that students need to work together
- Allow for argumentation in which students agree and disagree with one another, negotiate understanding, make claims supported by reasons and evidence, and reach consensus or agree where they disagree
- Have defined learning outcomes so students can connect task to specific learning
Where to Find Rich Tasks

• In your curricular materials!

• Illustrative Math: IllustrativeMathematics.org

• Inside Math: InsideMathematics.org

• Mathematics Assessment Project: map.mathshell.org

• Desmos Classroom Activities: desmos.com
Think - (Pair)- Share

The recommendation (from the Visible Learning authors) for deep learning is that students spend 50% of class time collaborating on and discussing rich math tasks.

➢ What are the benefits and challenges you anticipate with this recommendation?
Talk for Deep Learning

- Discourse is about the exchange of ideas.
- It reaches beyond discussion because it includes ways of representing, thinking, talking, agreeing and disagreeing.
Talk for Deep Learning

Students are able to:

- Ask clarifying questions
- Reason about which argument makes a stronger case
- Determine which strategy is more efficient
- Make an argument for when to use each strategy
- Identify, discuss & generalize patterns

Effect Sizes:

- Questioning – 0.48
- Classroom Discussion – 0.82
- Self-Verbalization & Self-questioning – 0.64
Talk for Deep Learning

89% of class time is teacher talk.

Students come to class to watch teachers do the work.

5-10% of teacher talk triggers more conversation or dialogue engaging students.
Accountable Talk

- Teachers and students share accountability for keeping the discussion on topic, presenting accurate information, and thinking deeply about the math.
- “A set of expectations for students that is supported through the use of language frames that scaffold the use of language to support a topic.”
## Accountable Talk Moves

<table>
<thead>
<tr>
<th>Move</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Press for clarification and explanation</strong></td>
<td>• Could you describe what you mean?</td>
</tr>
<tr>
<td></td>
<td>• Can you provide an example that supports your claim?</td>
</tr>
<tr>
<td></td>
<td>• Can you tell me more about your thinking about . . . ?</td>
</tr>
<tr>
<td><strong>Require justification of proposals and challenges</strong></td>
<td>• Where did you find that information?</td>
</tr>
<tr>
<td></td>
<td>• How did you know that?</td>
</tr>
<tr>
<td></td>
<td>• How does that support your claim?</td>
</tr>
<tr>
<td><strong>Recognize and challenge misconception</strong></td>
<td>• I don’t agree because . . .</td>
</tr>
<tr>
<td></td>
<td>• Have you considered an alternative such as . . . ?</td>
</tr>
<tr>
<td></td>
<td>• I think that there is a misconception here, specifically . .</td>
</tr>
<tr>
<td><strong>Require evidence for claims and arguments</strong></td>
<td>• Can you give me an example?</td>
</tr>
<tr>
<td></td>
<td>• Where did you find that information?</td>
</tr>
<tr>
<td></td>
<td>• How does this evidence support your claim?</td>
</tr>
<tr>
<td><strong>Interpret and use each other’s statements</strong></td>
<td>• David suggested . .</td>
</tr>
<tr>
<td></td>
<td>• What I heard Marla say was . .</td>
</tr>
<tr>
<td></td>
<td>• I was thinking about Jackson’s idea and I think . . .</td>
</tr>
</tbody>
</table>
Hopes & Dreams for Today

- Surface Learning – continued
  - Talk
  - Representation

- Deep Learning
  - Tasks
  - Talk (part I)
Learning from Mistakes: Linear Equations
Grade 8-12 / Math / Equations

- [https://www.teachingchannel.org/videos/students-learn-from-mistakes-ccssmdc](https://www.teachingchannel.org/videos/students-learn-from-mistakes-ccssmdc)

- **Optional:** Watch the Teaching Channel video “Learning from Mistakes: Linear Equations”

- HS lesson; Notice the use of collaboration, worked examples, critical thinking, determining efficient strategies, guided questions...all bridging from surface to deep learning
Think - (Pair) - Share

What were some takeaways from today?
THANKS!

See you Mar 27th!😊