Sequenced Units for the Common Core State Standards in Mathematics
Grade 3

In the years prior to Grade 3 students gained an understanding of number and used strategies based on place value, properties of operations, and the relationship between addition and subtraction to add and subtract within 1000. They worked with standard units of measure for length and described attributes of shapes.

Two major emphases of the Grade 3 year are the operations of multiplication and division and the concept of fractions. These concepts are introduced early in the year in order to build a foundation for students to revisit and extend their conceptual understanding with respect to these concepts as the year progresses. By the end of the year, students recall all products of two single-digit numbers. Third grade students develop understanding of fractions as numbers, and compare and reason about fraction sizes. This work with fractions is a cornerstone for developing reasoning skills and conceptual understanding of fraction size and fractions as part of the number system throughout this year and their future work with fractions and ratios. To continue the study of geometry, students describe and analyze shapes by their sides, angles, and definitions. In the final unit in this sequence of units, students generalize and apply strategies for computational fluency.

This document reflects our current thinking related to the intent of the Common Core State Standards for Mathematics (CCSSM) and assumes 160 days for instruction, divided among 15 units. The number of days suggested for each unit assumes 45-minute class periods and is included to convey how instructional time should be balanced across the year. The units are sequenced in a way that we believe best develops and connects the mathematical content described in the CCSSM; however, the order of the standards included in any unit does not imply a sequence of content within that unit. Some standards may be revisited several times during the course; others may be only partially addressed in different units, depending on the focus of the unit. Strikethroughs in the text of the standards are used in some cases in an attempt to convey that focus, and comments are included throughout the document to clarify and provide additional background for each unit.

Throughout Grade 3, students should continue to develop proficiency with the Common Core's eight Standards for Mathematical Practice:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

These practices should become the natural way in which students come to understand and do mathematics. While, depending on the content to be understood or on the problem to be solved, any practice might be brought to bear, some practices may prove more useful than others. Opportunities for highlighting certain practices are indicated in different units in this document, but this highlighting should not be interpreted to mean that other practices should be neglected in those units.

When using this document to help in planning your district's instructional program, you will also need to refer to the CCSSM document, relevant progressions documents for the CCSSM, and the appropriate assessment consortium framework.

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Grade 3

Unit 1: Exploring equal groups as a foundation for multiplication and division	Suggested number of days: 10

In Grade 2 students have added groups of objects by skip-counting and using repeated addition (**2.OA.C.4**). In this unit students connect these concepts to multiplication and division by interpreting and representing products and quotients.

Students begin developing these concepts by working with numbers with which they are more familiar, such as 2s, 5s, and 10s, in addition to numbers that are easily skip counted, such as 3s and 4s. Since multiplication is a critical area for Grade 3, students will build on these concepts throughout the year, working towards fluency by the end of the year.

Common Core State Standards for Mathematical Content	Comments
Operations and Algebraic Thinking — 3.OA A. Represent and solve problems involving multiplication and division. 1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.* 2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 × 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* 3. Use multiplication and division ~~within 100~~ to solve word problems in situations involving equal groups, arrays, ~~and measurement quantities~~, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. [1] NOTE: [1]See Glossary, Table 2. C. Multiply and divide within 100. 7. ~~Fluently~~ multiply and divide ~~within 100~~, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 × 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	In **3.OA.A.1** situations with discreet objects should be explored first when developing a conceptual understanding of multiplication, followed by measurement examples involving area models.[1] **3.OA.A.2** will be readdressed in unit 7 in order to provide students the opportunity to develop computational strategies as they extend the range of numbers with which they compute. **3.OA.A.3** will be readdressed in unit 7 and finalized in unit 14 to include measurement quantities in order to provide students multiple opportunities to develop and practice these concepts. **3.OA.C.7** will be readdressed in unit 7 and unit 15 in order to provide students the opportunity to develop computational strategies as they extend the range of numbers with which they compute.
Common Core State Standards for Mathematical Practice 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics.	Students use concrete objects or pictures to help conceptualize and solve problems (**MP.1**). They use arrays and other representations to model multiplication and division (**MP.4**) and contextualize given expressions (**MP.2**).

[1] For additional information see page 24 in the Operations and Algebraic Thinking progressions document.

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Unit 2: Developing conceptual understanding of area	Suggested number of days: 12
This unit provides ample time, and should include multiple experiences, for students to explore the connections among counting tiles, skip counting the number of tiles in rows or columns, and multiplying the side lengths of a rectangle to determine area. Students' understanding of these connections is critical content at this grade, and must occur early in the school year, thereby allowing time for understanding and fluency to develop across future units.[2]	
Common Core State Standards for Mathematical Content **Operations and Algebraic Thinking — 3.OA** B. Understand properties of multiplication and the relationship between multiplication and division. 5. Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.)* ~~*3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*~~ Note:[2] Students need not use formal terms for these properties. **Measurement and Data — 3.MD** C. Geometric measurement: understand concepts of area and relate area to multiplication and to addition. 5. Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of n square units. 6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). 7. Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. **Common Core State Standards for Mathematical Practice** 2. Reason abstractly and quantitatively. 6. Attend to precision. 7. Look for and make use of structure.	**Comments** **3.OA.B.5** will be readdressed in unit 9 with a focus on the distributive property and in unit 12 with a focus on the associative property. Students analyze the structure of multiplication and division (**MP.7**) through their work with arrays (**MP.2**) and work towards precisely expressing their understanding of the connection between area and multiplication (**MP.6**).

[2] For additional perspectives on the sequencing and placement of this content see page 16 of the PARCC Model Content Frameworks.

Sequenced Units for the Common Core State Standards in Mathematics
Grade 3

Unit 3: Developing strategies for addition and subtraction	Suggested number of days: 12
In Grade 2 students used addition and subtraction within 1000 using concrete objects and strategies. In this unit students increase the sophistication of computation strategies for addition and subtraction that will be finalized by the end of the year. This unit introduces the concept of rounding, which provides students with another strategy to judge the reasonableness of their answers in addition and subtraction situations. Perimeter provides a context in which students can practice both rounding and addition and subtraction (e.g. estimating the perimeter of a polygon).	
Common Core State Standards for Mathematical Content	**Comments**
Number and Operations in Base Ten — 3.NBT A. Use place value understanding and properties of operations to perform multi-digit arithmetic.[4] 1. Use place value understanding to round whole numbers to the nearest 10 or 100.	**3.NBT.A.1** introduces the concept of rounding, which is new to students and will be revisited in unit 8 in the context of multiplication.
2. ~~Fluently~~ add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. NOTE: [4]A range of algorithms may be used.	**3.NBT.A.2** will be finalized in unit 15 in order to give students time to reach fluency in addition and subtraction within 1000 by the end of the year.
Measurement and Data — 3.MD D. Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures. 8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length~~, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.~~	**3.MD.D.8** is the first time perimeter appears in the CCSS-M. Students are not expected to use formulas until Grade 4 (**4.MD.A.3**). **3.MD.D.8** will be addressed in full in unit 13 after students have been introduced to and worked with the concept of area.
Common Core State Standards for Mathematical Practice	
6. Attend to precision.	
8. Look for and express regularity in repeated reasoning.	Students use precise language to make sense of their solution in the context of a problem and the magnitude of the numbers (**MP.6**). Students also generalize algorithms and strategies and look for shortcuts (**MP.8**).

Sequenced Units for the Common Core State Standards in Mathematics
Grade 3

Unit 4: Understanding unit fractions	Suggested number of days: 10
In previous grades students have had experience partitioning shapes into fair shares (**1.G.A.3** and **2.G.A.3**), using words to describe the quantity. In this unit students extend this understanding to partition shapes and number lines, representing these fair shares using fraction notation. Similar to how students view 1 as the building block of whole numbers, students learn to view unit fractions as building blocks—understanding that every fraction is a combination of unit fractions.[3]	
Common Core State Standards for Mathematical Content	**Comments**
Geometry — 3.G A. Reason with shapes and their attributes. 2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*	The focus of **3.NF.A.1** and **3.NF.A.2a** in this unit is on fractions between 0 and 1. Fractions greater than 1 will be introduced in unit 5.
Number and Operations—Fractions[5] — 3.NF A. Develop understanding of fractions as numbers. 1. Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b. 2. Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. NOTE: [5] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.	Students use number lines to represent fractions in a new way (**MP.4**). It is key for students to have meaningful conversations around this concept to develop precise language about the components of fractions and location on the number line (**MP.3**, **MP.6**).
Common Core State Standards for Mathematical Practice 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 6. Attend to precision.	

[3] For additional perspectives on unit fractions, see the page 3 in the Fractions progressions document.

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Unit 5: Using fractions in measurement and data	Suggested number of days: 12
In this unit students extend their work with measurement and data involving whole numbers to include fractional quantities. Measurement and data are used as a context to support students' understanding of fractions as numbers. In students' work with data, context is important, because data are not just numbers; they are numbers with meaning. Through experience with measurement, students realize fractions allow us to represent data much more accurately than just representing data with whole numbers.[4]	
Common Core State Standards for Mathematical Content	**Comments**
Number and Operations—Fractions[5] — 3.NF A. Develop understanding of fractions as numbers. 1. Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b. 2. Understand a fraction as a number on the number line; represent fractions on a number line diagram. b. Represent a fraction *a/b* on a number line diagram by marking off *a* lengths *1/b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line. NOTE: [5] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8. **Measurement and Data — 3.MD** B. Represent and interpret data. 4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters. **Common Core State Standards for Mathematical Practice** 2. Reason abstractly and quantitatively. 5. Use appropriate tools strategically.	**3.NF.A.1** is repeated here to include fractions greater than 1. Students use tools to generate measurement data (**MP.5**) and make connections among different representations of the quantities and their relation to the given data context (**MP.2**).

[4] For additional information see pages 3-4 of the Categorical Data progressions document.

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Unit 6: Solving addition and subtraction problems involving measurement	Suggested number of days: 10
The focus of this unit is to develop a conceptual understanding of measuring mass, liquid volume, intervals of time, and using measurement as a context for the development of fluency in addition and subtraction.	
Common Core State Standards for Mathematical Content **Measurement and Data — 3.MD** A. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. 1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. 2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). [6] Add, subtract, ~~multiply, or divide~~ to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. [7] NOTE: [6] Excludes compound units such as cm3 and finding the geometric volume of a container. [7] Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2). **Common Core State Standards for Mathematical Practice** 1. Make sense of problems and persevere in solving them. 5. Use appropriate tools strategically. 4. Model with mathematics.	**Comments** **3.MD.A.1** is included here as an opportunity to model addition and subtraction situations with time as the context. **3.MD.A.2** is addressed in full in unit 14 to include multiplication and division situations. Students can apply the mathematics they know to persevere in solving problems arising in everyday life, society, and the workplace (**MP.1**, **MP.4**). Selecting and using appropriate tools supports the development of measurement concepts by asking students to reason about which tools are appropriate and how to use tools efficiently (**MP.5**).

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Unit 7: Understanding the relationship between multiplication and division	Suggested number of days: 12
The emphasis of this unit is for students to develop a solid understanding of the connection between multiplication and division. Students recognize that multiplication strategies can be used to make sense of and solve division problems. This unit provides students a solid foundation in solving problems with equal groups and arrays, which is necessary to support future success with measurement problems.[5]	
Common Core State Standards for Mathematical Content **Operations and Algebraic Thinking — 3.OA** A. Represent and solve problems involving multiplication and division. 2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 × 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* 3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1] NOTE: [1]See Glossary, Table 2. B. Understand properties of multiplication and the relationship between multiplication and division. 6. Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.* C. Multiply and divide within 100. 7. ~~Fluently~~ multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 × 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. **Common Core State Standards for Mathematical Practice** 1. Make sense of problems and persevere in solving them. 7. Look for and make use of structure.	**Comments** **3.OA.A.2** and **3.OA.C.7** are revisited in this unit to extend the range of numbers to include all numbers within 100 when multiplying and dividing. **3.OA.A.3** includes equal groups, arrays, and area problem types. Note that multiplicative compare problems are introduced in Grade 4 (**4.OA.A.2**).[6] **3.OA.C.7** is finalized in unit 15. This gives students the opportunity to develop and practice strategies in order to achieve fluency by the end of the year. Students make sense of and solve various types of multiplication and division problems (**MP.1**) by using the relationship between the two operations (**MP.7**).

[5] See the progressions document, K-5, Operations and Algebraic Thinking, Table 3: Multiplication and Division Situations, p. 23.
[6] See Table 2 on page 89 of the Common Core State Standards for Mathematics for the different types of problem situations.

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Unit 8: Investigating patterns in number and operations	Suggested number of days: 10
The focus of this unit is for students to identify arithmetic patterns in order to develop a deeper understanding of number and number relationships. In subsequent units, students will use the understanding of pattern developed in this unit to strengthen their computational strategies and skills.	
Common Core State Standards for Mathematical Content	**Comments**
Operations and Algebraic Thinking — 3.OA D. Solve problems involving the four operations, and identify and explain patterns in arithmetic. 8. Solve two-step word problems using the four operations. ~~Represent these problems using equations with a letter standing for the unknown quantity.~~ Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3] NOTE: [3]This standard is limited to problems posed with whole numbers and having whole- number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations). 9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends*. **Number and Operations in Base Ten — 3.NBT** A. Use place value understanding and properties of operations to perform multi-digit arithmetic. [4] 1. Use place value understanding to round whole numbers to the nearest 10 or 100. 3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations. NOTE: [4]A range of algorithms may be used. **Measurement and Data — 3.MD** B. Represent and interpret data. 3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*	**3.OA.D.8** will be revisited in unit 15 to address the use of equations and letters for unknown quantities. **3.NBT.A.1** is revisited in this unit to give students opportunities to make sense of rounding in multiplication situations. Students examine patterns in arithmetic (**MP.7**) and discuss what they discover (**MP.3**).
Common Core State Standards for Mathematical Practice 3. Construct viable arguments and critique the reasoning of others. 7. Look for and make use of structure.	

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Unit 9: Developing strategies for multiplication and division	Suggested number of days: 12
The focus for this unit is developing a conceptual understanding of decomposing multiplication problems through the use of the distributive property and the concept of area. Students are not required to use the properties explicitly, but are encouraged to discuss this concept and use area diagrams to support their reasoning. [7]	
Common Core State Standards for Mathematical Content	**Comments**
Operations and Algebraic Thinking — 3.OA B. Understand properties of multiplication and the relationship between multiplication and division. 5. Apply properties of operations as strategies to multiply and divide. [2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.)* ~~*3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.)*~~ *Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* Note: [2]Students need not use formal terms for these properties.	**3.OA.B.5** will be revisited in unit 12 to address the associative property of multiplication. Students use area diagrams and tiling (**MP.5**) to model the distributive property and generalize this experience to calculations (**MP.7**, **MP.8**).
Measurement and Data — 3.MD C. Geometric measurement: understand concepts of area and relate area to multiplication and to addition. 7. Relate area to the operations of multiplication and addition. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.	
Common Core State Standards for Mathematical Practice 5. Use appropriate tools strategically. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	

[7] For more information see page 26 in the Operations and Algebraic Thinking progressions document.

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Unit 10: Understanding equivalent fractions	Suggested number of days: 10
In this unit students develop a conceptual understanding of equivalence. Multiple types of models and representations should be used to help students develop this understanding. Students will apply their understanding of equivalence in the next unit as they learn to compare fractions. Through repeated experience locating fractions on the number line, students will recognize that many fractions label the same point and use this to support their understanding of equivalency.[8]	
Common Core State Standards for Mathematical Content **Number and Operations—Fractions[5] — 3.NF** A. Develop understanding of fractions as numbers. 3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram*. NOTE: [5]Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8. **Common Core State Standards for Mathematical Practice** 4. Model with mathematics. 6. Attend to precision.	**Comments** [**3.NF.A.3**] The focus of this unit is around equivalence. Although the cluster heading includes comparison of fraction, fraction comparisons **(3.NF.A.3d)** will be addressed in unit 11. Students develop understanding of equivalence by modeling fractions (**MP.4**) and communicating their understanding of what it means for fractions to be equivalent (**MP.6**).

[8] For more information on students' work with fractions on the number line see pages 3-4 in the Fractions progressions document.

Sequenced Units for the Common Core State Standards in Mathematics
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Unit 11: Comparing fractions	Suggested number of days: 10
In this unit students build on their prior work with fractions to reason about fraction size and structure to compare quantities. This unit focuses on a single standard to provide time for students to develop conceptual understanding of fraction comparisons and practice reasoning about size. Students defend their reasoning and critique the reasoning of others using both visual models and their understanding of the structure of fractions. This reasoning is important to develop a solid understanding of fraction magnitudes.[9]	
Common Core State Standards for Mathematical Content **Number and Operations—Fractions[5] — 3.NF** A. Develop understanding of fractions as numbers. 3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. NOTE: [5] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8. **Common Core State Standards for Mathematical Practice** 3. Construct viable arguments and critique the reasoning of others. 5. Use appropriate tools strategically. 7. Look for and make use of structure.	**Comments** Students will use their understanding of structure (i.e., the role of the numerator and denominator) **(MP.7)** to reason about relative sizes of fractions (**MP.3**).[10] Students use various tools to justify their comparisons, paying particular attention to the same-sized wholes (**MP.5**).

[9] For more information on students' reasoning about fractions see the Fractions progressions document.
[10] See Table 2 in CCSS-M (p.89) for the different types of problem situations.

Unit 12: Solving problems involving area	Suggested number of days: 10
The focus of this unit is to use area as a context to further develop multiplicative thinking. In this work, students bridge between concrete and abstract thinking, and use strategies to solve problems. This includes solving problems involving rectangular areas by multiplying side lengths and solving for an unknown number in related multiplication and division equations.	
Common Core State Standards for Mathematical Content **Operations and Algebraic Thinking — 3.OA** A. Represent and solve problems involving multiplication and division. 4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ☐ ÷ 3, 6 × 6 = ?.* B. Understand properties of multiplication and the relationship between multiplication and division. 5. Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* NOTE: [2]Students need not use formal terms for these properties. **Measurement and Data — 3.MD** C. Geometric measurement: understand concepts of area and relate area to multiplication and to addition. 7. Relate area to the operations of multiplication and addition. b. Multiply side lengths to find areas of rectangles with whole- number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. **Common Core State Standards for Mathematical Practice** 2. Reason abstractly and quantitatively. 6. Attend to precision. 8. Look for and express regularity in repeated reasoning.	**Comments** **3.OA.B.5** introduces the associative property explicitly for the first time. This property is fundamental for developing higher-level computation strategies.[11] In unit 9, students used various strategies to solve area problems. In **3.MD.C.7b** students recognize that they can find area in real-world situations by multiplying side lengths—without necessarily using a rectangular array. Students move in and out of context to solve these types of problems (**MP.2**) and use their repeated experience with area models to recognize that area problems can be solved using multiplication (**MP.8**). Students also explain precisely how an array corresponds to an expression (**MP.6**).

[11] For more information about the different levels of computation strategies see page 26 in the Operations and Algebraic Thinking progressions document.

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Unit 13: Solving problems involving shapes	Suggested number of days: 10
The focus of this unit is reasoning with shapes and their attributes, including area and perimeter. The standards in this unit strongly support one another because perimeter, like area, is an attribute of shape. Prior work with area and perimeter allows students differentiate between the two measures in this unit.[12]	
Common Core State Standards for Mathematical Content **Measurement and Data — 3.MD** D. Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures. 8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. **Geometry — 3.G** A. Reason with shapes and their attributes. 1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. **Common Core State Standards for Mathematical Practice** 1. Make sense of problems and persevere in solving them. 3. Construct viable arguments and critique the reasoning of others. 7. Look for and make use of structure.	**Comments** **3.MD.D.8** is addressed in full in this unit and focuses on distinguishing between linear and area measures and examining their relationship. Students look for and make use of structure (**MP.7**) as they determine categories and subcategories of shapes by identifying and reasoning about their attributes. Students make conjectures involving the attributes and measures of shapes and analyze various ways of approaching problems (**MP.1**, **MP.3**)

[12] For more information about these two attributes see page 18 in the Geometric Measurement progressions document.

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Unit 14: Using multiplication and division to solve measurement problems	Suggested number of days: 10
This unit extends students' work in unit 6 to include multiplication and division to solve problems involving measurement quantities.	
Common Core State Standards for Mathematical Content	**Comments**
Operations and Algebraic Thinking — 3.OA A. Represent and solve problems involving multiplication and division. 3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. [1] NOTE: [1]See Glossary, Table 2.	**3.OA.A.3** includes the use of all of the problem types Table 2 in CCSSM except for multiplicative compare problems—which will be introduced in Grade 4.[13]
Measurement and Data — 3.MD A. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. 2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). [6] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. [7] NOTE: [6] Excludes compound units such as cm3 and finding the geometric volume of a container. [7]Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2).	Students use strategies for multiplication and division to conceptualize and solve measurement problems (**MP.1**, **MP.2**). Students select appropriate tools and justify their selection for measuring different quantities (**MP.5**).
Common Core State Standards for Mathematical Practice 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 5. Use appropriate tools strategically.	

[13] For more information and examples of the different problem types, see page 89 in the Common Core State Standards for Mathematics.

Sequenced Units for the Common Core State Standards in Mathematics
Grade 3

Unit 15: Demonstrating computational fluency in problem solving	Suggested number of days: 10
This is a culminating unit in which students focus on problem solving in order to demonstrate fluency with addition and subtraction to 1000 and demonstrate fluency for multiplication and division within 100.	
Common Core State Standards for Mathematical Content	**Comments**
Operations and Algebraic Thinking — 3.OA C. Multiply and divide within 100. 7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 × 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. D. Solve problems involving the four operations, and identify and explain patterns in arithmetic. 8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3] NOTE: [3]This standard is limited to problems posed with whole numbers and having whole- number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations). **Number and Operations in Base Ten — 3.NBT** A. Use place value understanding and properties of operations to perform multi-digit arithmetic.[4] 2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. NOTE: [4]A range of algorithms may be used.	**3.OA.D.8** was introduced in unit 8 and is finalized in this unit to include the use of letters to represent unknown quantities in equations. Students demonstrate fluency in multiplication and division within 100 using various strategies and the properties of these operations (**MP.8**). They also represent these calculations and problem situations abstractly using letters (**MP.2**).
Common Core State Standards for Mathematical Practice 2. Reason abstractly and quantitatively. 8. Look for and express regularity in repeated reasoning.	